A (Co)inductive System Calculus for Security Properties

[New title suggestions are welcome!]

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ESORICS 2015 - PhD Symposium
October 2, 2015
Enforcement

Let $Sys$ be a set of systems.
Let $P : Sys \rightarrow \{ \text{FALSE, TRUE} \}$ be a system property.

**Definition**

A **sound enforcer** of $P$ is a mechanism $enf_P : Sys \rightarrow Sys$ such that, for all $\sigma \in Sys$, $enf_P(\sigma)$ satisfies $P$.

**Definition**

An enforcer $enf_P$ is **transparent** if and only if whenever $\sigma$ satisfies $P$, then $enf_P(\sigma) = \sigma$. 
Enforcement

Relevant questions:

- What is $Sys$?
- Sound and transparent enforcer for all properties?

Usually:

- Systems: C, JavaScript, automata, hardware, etc.
- Properties: not vulnerable to $\nu$, confidentiality, etc.

Know the power of your enforcer
Enforcing via Equations: An Artificial Toy Example

Consider the following

- Let $\text{Sys} = \mathbb{R} \rightarrow \mathbb{R}$
- Let $P : \text{Sys} \rightarrow \{ \text{FALSE}, \text{TRUE} \}$ defined, for $f \in \text{Sys}$, by
  \[ P(f) = f(r) \geq 0, \quad \text{for all } r \in \mathbb{R}. \]
- Let $| \cdot | : \text{Sys} \rightarrow \text{Sys}$ defined, for $f \in \text{Sys}$, by
  \[ |f|(r) = \begin{cases} f(r), & \text{if } f(r) \geq 0; \\ -f(r), & \text{otherwise}; \end{cases} \]

The function $| \cdot |$ is one sound and transparent enforcer for $P$.
Enforcing via Equations: An Artificial Toy Example

Your competition proposes

\[
\text{enf}_P(f)(r) = \begin{cases} 
  f(r), & \text{if } f(r) \geq 0; \\
  0, & \text{otherwise};
\end{cases}
\]

- Enforcement policy: use \( \text{enf}_P \) or \(|\cdot|\)?

Enforcement: not only about what, but also about how.

Verifying vs. enforcing

- Verify: prove \( f(r) \geq 0 \) for all \( r \in \mathbb{R} \) (maybe hard).
- Enforce: use \(|f|\) or \( \text{enf}_P \) instead of \( f \) (easy)
Motivation

It would be nice if we could do the same for complex systems and for practical security properties

Can we actually do this?
Motivation

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Can we actually do this?

Hopefully yes, using coinductive calculus.
Before we continue

I will try to convince you that...

- Coinduction: break systems apart, rebuild them back.
- Enforcement: rebuild systems so they satisfy a property.
- Implementation: equations lazily evaluated in Haskell.
Coinduction: Breaking Streams Apart

**Streams** (Single-threaded, non-interactive systems)

- Let $\mathbb{R}^\omega = \{ [r_0, r_1, \ldots] \mid r_i \in \mathbb{R} \}$
- Let $\text{head}: \mathbb{R}^\omega \to \mathbb{R}$ defined by
  \[
  \text{head}( [r_0, r_1, \ldots] ) = r_0.
  \]
- Let $\text{tail}: \mathbb{R}^\omega \to \mathbb{R}^\omega$ defined by
  \[
  \text{tail}( [r_0, r_1, \ldots] ) = [r_1, \ldots].
  \]

Stream $\sigma$ is *coinductively* defined by its *head* and *tail*
Coinduction: Rebuilding Streams from Pieces

Let \( \text{pack} : \mathbb{R} \times \mathbb{R}^\omega \rightarrow \mathbb{R}^\omega \) be defined by

\[
\text{pack}(r, [r_0, r_1, \ldots]) = [r, r_0, r_1, \ldots]
\]

\( \text{pack} \) is the “compiler” of the specification \( \langle r, [r_0, r_1, \ldots] \rangle \)

\[
\mathbb{R} \times \mathbb{R}^\omega \cong \mathbb{R}^\omega
\]

Modify the head and/or tail to obtain a different stream.

\[
\text{enf}_P(\sigma) = \text{pack}(f \circ \text{head}(\sigma), g \circ \text{tail}(\sigma))
\]
Another Toy Example

Define enforcers using head, tail and pack

Let \(|\cdot|: \mathbb{R}^\omega \rightarrow \mathbb{R}^\omega\) defined, for \(\sigma \in \mathbb{R}^\omega\) by

\[
|\sigma| = \begin{cases} 
\text{pack}(\langle \text{head}(\sigma), |\text{tail}(\sigma)| \rangle), & \text{if } \text{head}(\sigma) \geq 0; \\
|\text{tail}(\sigma)|, & \text{otherwise}; 
\end{cases}
\]  

(1)

| \cdot | soundly and transparently enforces “always \(\geq 0\)”

Equation (1) is a behavioural (differential) equation.
Let $X$ be a Haskell type implementing:

- **observe**: $X \rightarrow \mathbb{R}$
- **next**: $X \rightarrow X$

**Enforce “always $\geq 0$” on $X$ using $\cdot \mid$ by projecting $X$ into $\mathbb{R}^\omega$**
From Streams to Arbitrary Types

Let $X$ be a Haskell type implementing:

- \textbf{observe}: $X \rightarrow \mathbb{R}$
- \textbf{next}: $X \rightarrow X$

\textbf{Enforce “always $\geq 0$” on $X$ using $\lvert \cdot \rvert$ by projecting $X$ into $\mathbb{R}^\omega$}

Let $[\cdot] : X \rightarrow \mathbb{R}^\omega$ be defined, for $x \in X$, by

$$[x] = \text{pack}((\text{observe}(x), [\text{next}(x)])$$

$[x]$ satisfies “always $\geq 0$” and $x$ and $[x]$ are behaviourally equivalent.
Non-interference

Let $I$ be a set of inputs, $\text{lvl}: I \rightarrow \{ \mathcal{L}, \mathcal{H} \}$ be an input classification function, and $X$ be a Haskell type implementing:

- $\text{observe}: X \rightarrow I \rightarrow \mathbb{R}$ (an $\mathcal{L}$-channel)
- $\text{next}: X \rightarrow I \rightarrow X$

Non-interference: the presence of $\mathcal{H}$-actions does not impact $\mathcal{L}$-channels.

\[
\begin{align*}
\text{observe}(\text{enf}_P(\sigma), i) &= \begin{cases} 
\text{observe}(\sigma, i), & \text{if lvl}(i) = \mathcal{L}; \\
\varepsilon, & \text{otherwise}.
\end{cases} \\
\text{next}(\text{enf}_P(\sigma), i) &= \begin{cases} 
\text{enf}_P \circ \text{next}(\sigma, i), & \text{if lvl}(i) = \mathcal{L}; \\
\text{enf}_P(\sigma), & \text{otherwise}.
\end{cases}
\end{align*}
\]
Contribution

Illustrate how systems, properties and enforcement mechanisms can be brought down to the same abstraction level; resulting in a practical framework for the enforcement of security properties.
Objective

Find and solve systems of behavioural equations to obtain systems that satisfy security properties.

Milestones:

- Find equations that define security properties
  - Prove expressivity: “benchmark” properties
  - Classify properties according to enforceability
- Develop tool support: Haskell
Questions

Questions?

Thank you for your attention!